

PERFORMANCE OPTIMISATION OF COMPLEX DYNAMIC SYSTEMS USING NEURAL NETWORK

P.P.Mohanlal * Uma Syamkumar ** S.Dasgupta*

*Control and Guidance Design Group, VSSC, Trivandrum, India.

**Senior Research Fellow, Dept. of Computer Science, University of Kerala.

Abstract: Neural networks can be used effectively for the identification and control of non-linear dynamic systems. The emphasis of this paper is on extending the static back propagation algorithm to non-linear dynamic systems. The method termed dynamic back propagation is applied to a complex dynamic system particularly for adaptive control. An alternative over the dynamic back propagation has been dealt with which makes use of an adjoint model of the system. An adaptive optimisation scheme is studied using the above techniques to enhance the performance of a classical feedback control system design.

1. INTRODUCTION

The standard back propagation algorithm is well known for training feed forward neural networks to deal with static function mapping problems such as pattern recognition. This computationally efficient algorithm is used in many neural control applications. Neural control paradigm is widely recognised as useful for non-linear control problems with uncertainties. Recurrent neural networks have better representational capabilities than feed forward networks. Training of recurrent neural networks, considering it as a non-linear dynamic system is complex and computationally intensive. But, it becomes necessary to use recurrent neural networks to infer internal dynamics of systems when the relation between internal dynamics and measured outputs is not invertible. Also, while using even feed forward networks with interconnected dynamics, the training algorithms for the feed forward networks become that of an effective recurrent network. In this paper, two training methods, namely, the dynamic back propagation (DBP)[2] and the Adjoint back propagation (ABP) [3] for recurrent neural networks are used for the performance optimization of a general feedback control system using a neural control module with the conventional scheme. The sensitivity network for the proposed scheme is developed. Applicability of ABP for the proposed scheme is verified. Performance optimisation with a neural control block using DBP and ABP is achieved.

2. DYNAMIC BACKPROPAGATION FOR NONLINEAR DYNAMIC SYSTEMS

In a causal dynamic system, a change in a parameter at time, k , will produce a change in the output, $y(t)$ for all $t \geq k$. For example, given a non-linear dynamic system, $x(k+1) = \phi(x(k), u(k), \theta)$; $y(k) = \Psi(x(k))$(1) where θ is a parameter, u is the input and x is a state vector, then the partial derivative of $y(k)$ with respect to θ can be obtained by solving the following linear state equations,

$z(k+1) = A(k)z(k) + V(k)$; $z(0) = 0$; $w(k) = C(k)z(k)$.
where $z(k) = \delta x(k) / \delta \theta$; $A(k) = \phi_x(k)$; $V(k) = \phi_\theta(k)$;
 $w(k) = \delta y(k) / \delta \theta$; $C(k) = \Psi_x(k)$.

ϕ_x and Ψ_x are the Jacobian matrices and the vector ϕ_θ represents the partial derivatives of ϕ with respect to θ . In the above equations, if $A(k)$, $V(k)$ and $C(k)$ can be computed, $w(k)$ can be obtained as the output of the dynamic sensitivity model which is the linearised version of the original non-linear dynamic system around a nominal trajectory and input. Thus, the required gradient computation becomes possible for recurrent network which is a non-linear dynamic system. If there are M adjustable parameters, this method requires M separate sensitivity networks to be generated to get all the M gradient components of the vector. However, this is a forward method and is not the generalisation of static backpropagation.

3. ADJOINT BACKPROPAGATION FOR NON-LINEAR DYNAMIC SYSTEMS [3]

The co-state equations associated with any optimisation problem actually represent the adjoint of the original system. This is also known as the dual of the original system. The co-states are the states of the adjoint system. The adjoint mathematics is advantageously used

for sensitivity analysis for tactical missile guidance applications where the forward system is a linear time varying system[4]. Sensitivities are observed from the adjoint system to calculate the control for the forward system. Viewing the static backpropagation algorithm in this way, actually it uses the adjoint network of the forward network for computing the necessary gradients. Extending this idea to recurrent networks requires to unfold the network in time backwards for a finite duration for the construction of the adjoint, from which the necessary gradients of the forward network can be computed. The backpropagation through time (BTT) [5,6] is closely related to this idea.

Consider a non-linear dynamic system,
 $y(k) = f(y(k-1), y(k-2), \dots, y(k-n), u(k), u(k-1), \dots, u(k-m), W_i(k))$ is represented by a recurrent neural network whose parameters at time, $k = W_i(k)$.
 Let the scalar cost be $J(k) = \frac{1}{2} e(k)^T e(k)$
 Where $e(k) = y(k) - y_p(k)$; $y_p(k) =$ target o/p ; $y(k) =$ network o/p.

If the error is backpropagated through the adjoint model whose initial conditions are set to zero, with the adjoint input = $e(k)$ at time k and zero elsewhere, then, the adjoint states are given by

$\lambda(0) = e(k)$; $\lambda(i) = \sum_{y(k-i)}^T f(k-i+1) \lambda(i-1)$;
 $i=1, 2, \dots, \varphi$ and the gradients are given by
 $\nabla_{W_{\ell}(k-\varphi)} J(k) = \sum_{W_{\ell}(k-\varphi)}^T f(k-\varphi) \lambda(\varphi)$(2)
 where, $\sum_{y(k-i)}^T f(k-i+1)$ and $\sum_{W_{\ell}(k-\varphi)}^T f(k-\varphi)$ are the Jacobians obtainable from map $f(\cdot)$ at different time instances.

The gradient obtained $\nabla_{W_{\ell}(k-\varphi)} J(k)$ is the gradient of the current performance with respect to an earlier parameter. This can cause problems in the update algorithm if proper care is not taken. However, if we keep the parameters of the network constant for a certain finite duration, then the difficulty can be overcome by sacrificing some information which otherwise could have been used [3].

The method of obtaining the gradients using ABP is computationally advantageous compared to DBP since one single adjoint network gives all the required gradient components. Also, it is the true generalisation of the static back propagation.

4. NEURAL PERFORMANCE OPTIMISATION OF A CLASS OF FEEDBACK CONTROL SYSTEM USING DBP

In figure 1, NC is the neural controller. Without the NC, the figure:1 represents the auto-pilot configuration of a typical launch vehicle [8]. All the feedback and forward filters are usually well-designed for a nominal plant. The plant is a linearised version of a non-linear system. Usually there will be many uncertainties in the plant model and performance will not be as expected. Here, by using the NC block it is

attempted to track the output of a reference model. NC is a three layer network with tanh sigmoids in the hidden layers and linear activation in the output layer. There are 20 neurons in the second hidden layer, 10 neurons in the first hidden layer and 1 neuron in the output layer. There are 5 moving average inputs and 5 recurrent inputs.

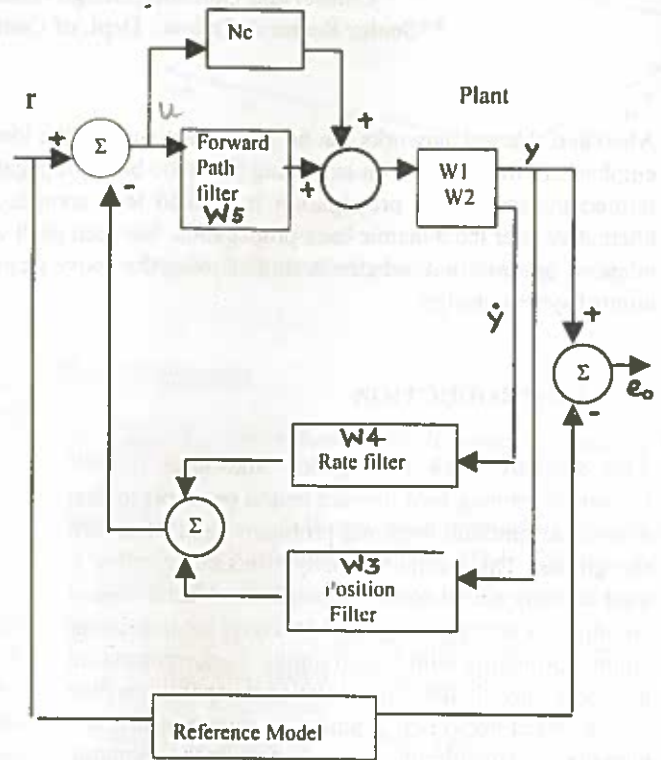


Fig.1

4.1 DEVELOPMENT OF THE SENSITIVITY NETWORK

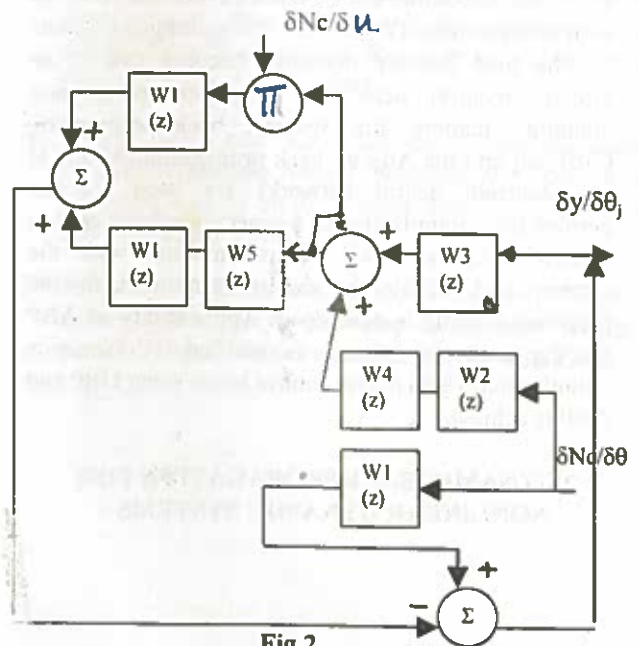


Fig.2

The above sensitivity network was developed for the system in Fig. 1, to get the $\delta y / \delta \theta_j$ where θ_j is a typical parameter of the NC. The reference model is a second order system with damping, 0.7 and natural frequency, 3 rad/sec. The rate and position filters are of the first order. The plant is of ninth order. The forward path filter is of fifth order.

Initially, the input, r , was chosen as a uniformly distributed random signal in the interval, (-1,1) and general training was carried out using the gradients obtained from the sensitivity network above. Once reasonable convergence was attained, a step input was applied and the step-response of the reference plant and the actual plant were plotted and are given in Fig. 3. Since the network size is large, the computational time involved is high. Further iterations are required to obtain perfect matching. The performance convergence clearly validates the sensitivity network and the optimisation scheme.

5. PERFORMANCE OPTIMISATION USING ABP

In the configuration shown in Fig. 1 it was attempted to use ABP for performance optimisation. Due to the particular configuration used here, the sensitivity network (Fig. 2) also had to be used to obtain $\nabla_{w(k-\varphi)}^T f(k-\varphi) = \nabla_{\theta(k-\varphi)}^T y(k-\varphi)$ which has to be further used in eqn. 2 along with the adjoint states, $\lambda(\varphi)$. The adjoint system was constructed by linearising NC each instant and unfolding the system upto the number of time-steps equal to the order of the overall system to get the adjoint states. Then the required performance gradients were computed using the algorithm given in section 3. The results of the optimisation for step input is given in Fig. 4. Here also further iterations are required for perfect matching.

6. DISCUSSIONS AND CONCLUSIONS

When neural control schemes are used for the performance optimisation of dynamic systems, either DBP or ABP or both have to be used for obtaining the gradient of the performance.

In the specific example illustrated, both these methods are used for optimisation to validate the sensitivity network developed and the construction of the adjoint system.

Even though convergence is confirmed, the convergence rate is slow mainly due to the large size of the NC block. The size of the neural block can be reduced provided the sub-blocks of Fig. 1 are reasonably

well-designed whereby convergence rate can be improved.

A typical performance optimisation scheme for the classical feedback control system is proposed and feasibility ensured. However, this scheme is not suitable if the plant is unstable and/or non-minimum phase in which case the sensitivity network becomes unstable. Alternate schemes are under study to deal with such cases.

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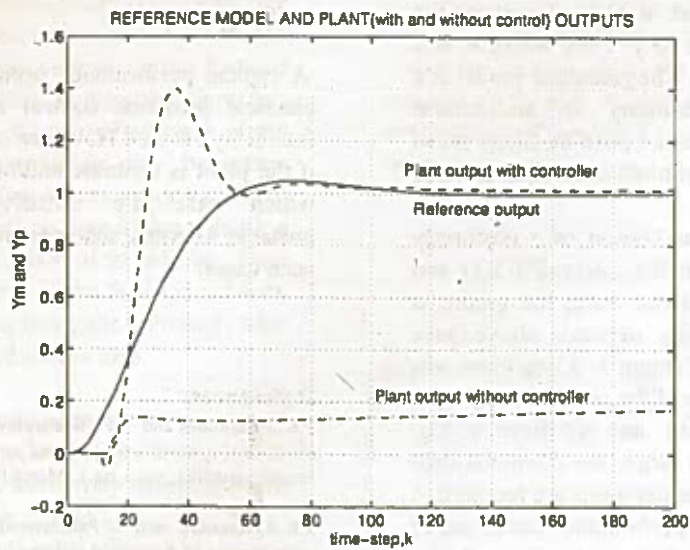


Figure 3

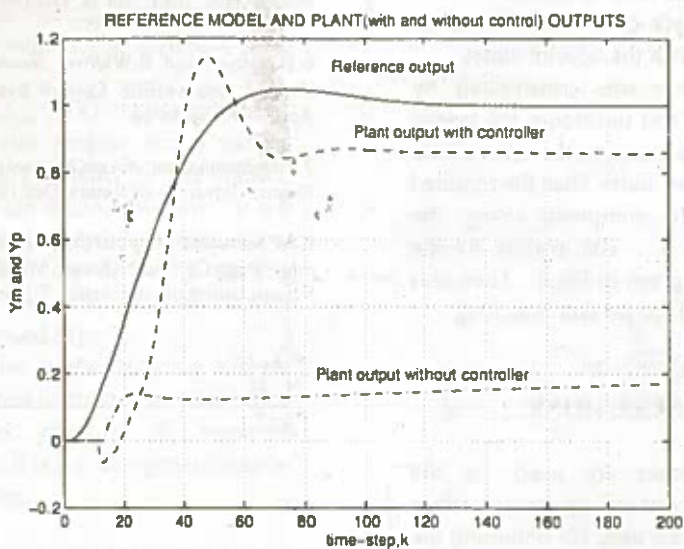


Figure 4